Addition to "MSE Analysis of a Multi-Loop LMS Pseudo-Random Noise Canceler for Mixed-Signal Circuit Calibration"

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The analysis presented in [1] restricts the adaptive filter input sequences, i.e., the $S_k[n]$ sequences in (1), to have magnitudes of either 0 or 1 for each k and n as described in the introduction. However, the results of the paper also hold if the magnitude of $S_k[n]$ is generalized to be any value between the range of 0 and 1. This generalization requires a change of the proof of Lemma B5. Specifically, the paragraph following Equation (135) in the proof of Lemma B5 should be replaced to the following paragraph to accommodate this generalization:

If $a_k[n] = S_k[n-Q]S_l[n-j]$ with $l \neq k$ or $j \neq Q$, then $E\{a'_k[n]\} = 0$ because $S_k[n-Q]$ and $S_l[n-j]$ are independent and $E\{S_k[n-Q]\} = 0$. If $a_k[n] = (S_k^2[n-Q]-c_k[n-Q])/2$, then $E\{a'_k[n]\} = 0$, because $c_k[n-Q] = E\{S_k^2[n-Q]\}$. For every other case, $a'_k[n]$ can be written as $a'''_k[n]S_k[n-Q]$, $a'''_k[n](S_k^2[n-Q]-c_k[n-Q])/2$, or $a'''_k[n]c_k[n-Q]$, where $|a'''_k[n]| \leq 1$. In the latter two cases where $a'_k[n]$ is $a'''_k[n](S_k^2[n-Q]-c_k[n-Q])/2$ or $a'''_k[n]c_k[n-Q]$, it follows from $|a'''_k[n]| \leq 1$ and $c_k[n-Q] = E\{S_k^2[n-Q]\}$ that $|E\{a'_k[n]\}| \leq c_k[n-Q]$. It remains to show that $|E\{a'_k[n]\}| \leq c_k[n-Q]$ also holds for $a'_k[n] = a'''_k[n]S_k[n-Q]$. This is proven as follows. Since $a'_k[n]$ by definition is the product of all $S_l[n-Q]$, $0.5(S_l^2[n-Q]-c_l[n-Q])$, and $c_l[n-Q]$ of any l in $a_k[n]$, it follows that $a'''_k[n]$ is either deterministic 1 or is also a product of $S_l[n-Q]$, $0.5(S_l^2[n-Q]-c_l[n-Q])$, and $c_l[n-Q]$ of any l. If $a'''_k[n]$ is deterministic 1 or is a product of $S_l[n-Q]$, $0.5(S_l^2[n-Q]-c_l[n-Q])$, and $c_l[n-Q]$ for $l \neq k$, it follows from $E\{S_k[n-Q]\} = 0$ and $S_k[n-Q]$ is uncorrelated with $S_l[n-Q]$ for $l \neq k$ that $|E\{a'_k[n]\}| = |E\{a'''_k[n]S_k[n-Q]\| = 0 \leq c_k[n-Q]$. Otherwise, $a'''_k[n]$ can be further written as $a'''_k[n] = b'''_k[n]c'''_k[n]$, where $b'''_k[n] \in \{S_k[n-Q], 0.5(S_k^2[n-Q]-c_k[n-Q]), c_k[n-Q]\}$ and $|c'''_k[n]| \leq 1$, thus $|E\{a'_k[n]\| = |E\{a'''_k[n]S_k[n-Q]\}| = |E\{b'''_k[n]S_k[n-Q]c'''_k[n]\}| \leq E\{|b'''_k[n]S_k[n-Q]|\}$, and it follows from the definition of $b'''_k[n]$ above that $|E\{a'_k[n]\| \leq c_k[n-Q]$ also holds. Therefore, $|E\{a'_k[n]\}| \leq \rho c_k[n-Q]$ where ρ is as defined in the lemma statement. Substituting this into (135) and applying Lemma C2 yields

1. D. Kong and I. Galton, "MSE Analysis of a Multi-Loop LMS Pseudo-Random Noise Canceler for Mixed-Signal Circuit Calibration," *IEEE Trans. Circuits Syst. I:Reg. Papers*, 2020.