# Correlation Techniques for Estimation of Amplifier Nonlinearity

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***Abstract* — Nonlinearity characterization is critical to amplifier linearization. It is shown here that correlation techniques can be used to provide an estimate of amplifier nonlinearity. The output envelope is cross-correlated with a test sequence generated by forming the product of multiple uncorrelated input sequences, to yield estimates of the low order nonlinearity coefficients. The proposed method can be employed to estimate amplifier nonlinearity with IS-95 forward-link CDMA signals, in background during amplifier operation.**

***Index Terms* — Correlation, power amplifiers, nonlinear estimation.**

## I. Introduction

Power amplifier linearity is a critical characteristic in many digital communication systems, since amplifier nonlinearity can lead to generation of out-of-band power components, or reduction of the accuracy of in-band signals. Accurate amplifier characterization is a first important step for amplifier linearization, frequently followed by the use of input predistortion. Traditional nonlinearity measurements are done with single tones of swept power (to provide AM-AM and AM-PM distortion characteristics), or by use of two-tone tests to provide measurements of third order intermodulation products [1-2]. Based on the extracted behavioral model, statistical techniques can be used to predict spectral regrowth with complex digital modulation [3-5]. These approaches require special input signal generation, and are generally not applicable for characterization of amplifiers during actual operation. In this paper, we demonstrate simple correlation techniques to estimate power amplifier nonlinearity. The IS-95 forward link (base station to mobile) CDMA signal is employed as an example. First an appropriate test sequence is generated from multiple input data sequences. Then the cross-correlation of this test signal and the baseband output signal is calculated to estimate the nonlinearity. The method provides an estimate of the low order coefficients of nonlinearity around a given operating power, in a manner similar to the measurement of intermodulation products such as IM3. However, the technique provides both real and imaginary parts of the nonlinearity coefficients. Because the test signal is inherently generated from the actual CDMA signal, this method can be carried out without interrupting the normal amplifier operation. It can also potentially be carried out with simple circuitry.

## II. Correlation Techniques for Nonlinearity Estimation

In the following, it is assumed that the amplifier is a quasi-memoryless system (although memory effects can potentially be included in a direct fashion). To describe the amplifier within the bandpass nonlinear model, we use an odd order complex power series

$$\tilde{V}_{out} = G(|V_{in}|)\tilde{V}_{in} = \tilde{V}_{in}(\tilde{\alpha}_1 + \tilde{\alpha}_3|V_{in}|^2 + \tilde{\alpha}_5|V_{in}|^5) \quad (1)$$

where $G(|V_{in}|)$ is the nonlinear gain function, which is a polynomial function of the envelope of the input signal and $\tilde{a}_1$, $\tilde{a}_3$ and $\tilde{a}_5$ are general complex coefficients. In (1), only the odd terms are kept, which produce intermodulation in band and adjacent channel distortions. In this paper, the polynomial is truncated to the $5^{th}$ order. Including higher orders will provide better accuracy, but will need more computational time and provide larger relative estimation error in this approach, which will be discussed in subsession *B*. Moreover, assessment of the lowest order nonlinearity terms is of primary importance to improve linearity [6].

### *A. Analytical formulation*

Since the nonlinearity comes from the output's third and fifth order dependency on the input, the nonlinearity can be extracted by the correlations of output and the generated test sequences, which is also third and fifth order function of input. The basic approach is shown in Fig.1. For simplicity, here we assume all polynomial coefficients and the input signal are real. $S_1$, $S_2$ *and* $S_3$ are three PN (pseudonoise) sequences with amplitude of ± 1 and they are uncorrelated with each other. The summation of them is upconverted to the carrier frequency to provide the PA's input. A test signal $S_{test}$ is created, which is the product of the input sequences. The output signal, which is a nonlinear transform of input signal, is down-converted to the baseband and correlated with the $S_{test}$ signal to get the estimates of nonlinearity. The number of the

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sequences can be increased to extract higher order nonlinearity; three sequences are used here for analysis simplicity, although our measurements used five sequences to obtain contributions to fifth order.

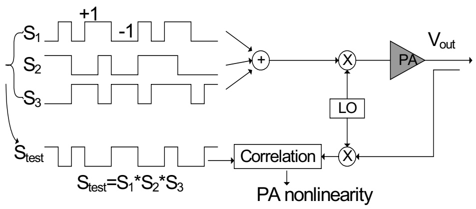


Fig. 1. Correlation methodology

Let us denote the input signal as a summation of the three PN sequences $S_1 \sim S_3$.

$$V_{in}(t) = \sum\left(S_1(t) + S_2(t) + S_3(t)\right) \tag{2}$$

It can be seen that in the expansion of (1), the 3rd and 5th order terms $V_{in}(t)^3$ and $V_{in}(t)^5$ will generate expended terms like $\sum_{n=1}^{N} C_n S_1(t)^i S_2(t)^j S_3(t)^k$, where *i, j* and *k* are odd integer numbers with constraints $i+j+k=3$ or $i+j+k=5$ and $C_n$ is numerical constant. $S_1^i S_2^j S_3^k$ are called $S_{test}$ terms and the remain are *non*-$S_{test}$ terms denoted by $\overline{S_{test}}$.

The time average cross-correlation function of $V_{out}$ and $S_{test}$ is defined as

$$\Phi_{Vout,Stest}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T V_{out}(t) S_{test}(t+\tau)dt \tag{7}$$

The right part (7) can be written as

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \left[ \underbrace{(a_3 C_1 + a_5 \sum_{n=1}^{N} C_n)}_{\overline{a}_3} S_{test}(t) + \overline{S_{test}} \right] S_{test}(t+\tau)dt \tag{8}$$

where $\overline{a}_3$ is called equivalent nonlinear coefficient, which is a combination of $a_3$ and $a_5$ here. Using PN sequences properties, the maximal cross-correlation value occurs at zero offset position.

$$\Phi_{Vout,Stest}(0) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[(\overline{a}_3 S_{test}(t)^2 + otherterms\right]dt \tag{9}$$

The only dominant term left in (9) is the square of $S_{test}$ term; other terms will be averaged to zero. The maximal value is proportional to $\overline{a}_3$ . In practical wireless transmitters, the PN sequences $S_1$, $S_2$ and $S_3$ will pass through a linear waveform-shaping filter before upconversion. The analysis is very similar to the case without filter. For example, the cubic of the filtered input signal $V_{in\_f}$ can be written as

$$V_{in\_f}(t)^3 = \left(\sum_{i=0}^{K-1} V_{in}(t-i)u_i\right)^3 \tag{10}$$

where K is number of taps of the FIR (finite impulse response) filter and $u_i$ (*i* from 1 to K-1) are the filter's coefficients, of which the maximal one is assumed to be $u_m$. So the maximal cross-correlation value occurs at a non-zero offset position.

$$\begin{aligned} &Max3rd(\Phi_{Vout,Stest}(\tau)) = \Phi_{Vout,Stest}(-m) \\ &= \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[u_m^3 \overline{a}_3 S_{test}(t-m)^2 + otherterms\right]dt \end{aligned} \tag{11}$$

*B. Simulation results of polynomial PA and error analysis*

Matlab simulation setup shown in Fig.2 was used to verify this methodology.

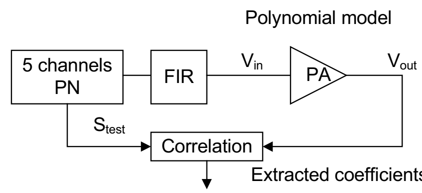


Fig. 2. Simulation setup

Five PN sequences generated by the random functions in Matlab were filtered by the IS-95 48-tap FIR filter [7]. To generate the distorted output signal, the input was multiplied by a polynomial PA model, which is a polynomial function with fixed real coefficients, shown in the first column of Table 1. The ideal and correlation extracted equivalent coefficients are shown in column 2 to 8 for cases without and with filter. The simulation results show the estimation accuracy depends on heavily sequence length *L*. The reason is that the PN sequences used in practice are not ideal, but exhibit some correlation. It is known that the cross-correlation between any pair of binary sequence has a lower bound developed by Welch [8]. Two types of error are defined here to analyze the estimation accuracy. The inter-sequence partial correlation error shown in (12)

$$\sum_{i=1}^{L} (S_1[i] S_2[i]) / L = \sigma_1 \tag{12}$$

denotes the non-zero cross-correlation of two uncorrelated PN sequence $S_1$ and $S_2$ . The intra-sequence partial correlation defined in (13)

$$\sum_{\substack{i,j=1 \\ i\neq j}}^{L} (S_1[i] S_1[j]) / L = \sigma_2 \tag{13}$$

denotes the non-zero autocorrelation of PN sequence $S_1$ at non-zero offset. Fig. 3 shows the absolute values of normalized partial correlation errors vs. sequence length.

Longer sequence is required to achieve better estimation accuracy.

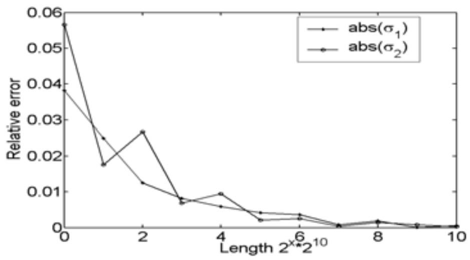


Fig.3. Inter and intra-sequence partial correlation error

In (11), the otherterms are not averaged to zero any more, but to some estimation errors due to the partial correlation of PN sequences. In Table 1, the estimate accuracy of the case with filter is poorer than the case without filter. The reason is that more otherterms can contribute estimation errors in the former case. And in Table 1, the relative estimation error increases as the order increases. The mainly reason is that the initial polynomial coefficients of higher order are smaller than lower order, so the maximal correlation values of higher order are smaller than lower order, which causes that the estimate accuracy of higher order can be largely affected by the nonideal estimation errors. This is one of the reasons why only 5[th] order polynomial modeling is used.

*C. Complex input and simulation results of LUT PA*

The analysis of previous session can be extended to complex baseband input signal case. The input signal in its rectangular format is

$$\widetilde{V}_{in}(t) = I(t) + jQ(t) \tag{14}$$

where inphase signal $I$ and quadrature signal $Q$ are also summations of uncorrelated PN sequences. Substituting (14) into (1), the in-phase part of output signal can be written as

$$V_{out\_I} = a_{3r}I^3 + 2a_{5r}Q^2I^3 + a_{5r}I^5 - (a_{3i}Q^3 + 2a_{5i}I^2Q^3 + a_{5i}Q^5) \tag{15}$$

where only order higher than or equal to three are kept for nonlinearity extraction purpose. Using similar analysis in session *A,* the equivalent real and imaginary part of coefficients can be extracted by correlation between $V_{out\_I}$ and in-phase or quadrature $S_{test}$ signal shown in (16)

$$\text{Max3rd}(\phi_{\text{Vout\_I, Stest\_I}}(\tau)) = C_1a_{3r} + C_2a_{5r} \propto \overline{a}_{3r} \tag{16}$$
$$\text{Max3rd}(\phi_{\text{Vout\_I, Stest\_Q}}(\tau)) = C_1a_{3i} + C_2a_{5i} \propto \overline{a}_{3i}$$

And the power correlation value is defined in (17) to study its changing trend versus input power next.

$$\text{Max3rd}(\phi_{\text{Vout\_I, Stest}}(\tau))^2 = \text{Max}(\phi_{\text{Vout\_I, Stest\_I}}(\tau))^2 + \text{Max}(\phi_{\text{Vout\_I, Stest\_Q}}(\tau))^2 \propto \left|\widetilde{\overline{a}}_3\right|^2 \tag{17}$$

The simulation setup is same as Fig.2 except that a LUT (lookup-table) PA model was used and the table data came from the measurement of Intersil's ISL3990 Dual Band GaAs PA. The AM-AM and AM-PM characteristics are shown in Fig.4. The $P_{1db}$ compression point is at –10.5 dBm input power.

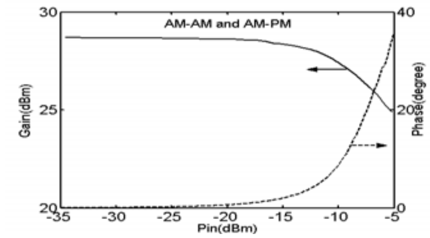


Fig.4. AM_AM and AM_PM characteristics

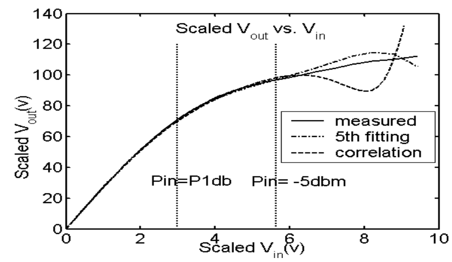


Fig.5. Scaled output magnitude vs. input magnitude

The solid curve in Fig.5 is the scaled output magnitude versus input magnitude based on previous measurement. The dash-dot curve is the five-order polynomial fitting of the measured data. And the dash curve shows the estimation results using polynomial model with coefficients extracted by correlation. The phase shift versus input magnitude is given in Fig.6. The estimation was carried on when the average input power of CDMA signal is at $P_{1db}$. These two figures show the curves matching are pretty good in the range of very low to –5 dBm input power, but poor at higher power. The reason is that the occurrence of very high power is of very low probability as shown in Fig.7 and the estimated coefficients can only cover the finite input power range.

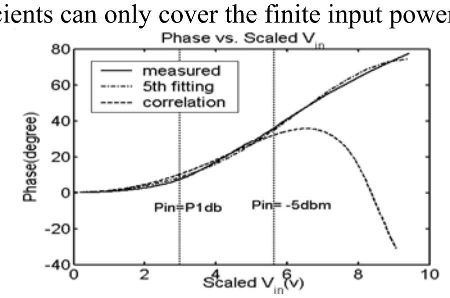


Fig.6. Phase shift vs. input magnitude

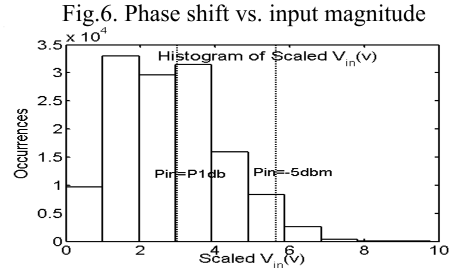


Fig.7. Histogram of input magnitude

The simulated power correlation values versus input power are shown in Fig.8. The slope of these curves is

1:3:5 with respect to $P_{in}$. At lower input power, the 5th order nonlinearity is very small and its estimate accuracy is poor due to the effect of estimation errors.

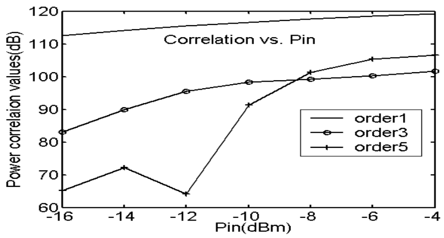


Fig.8. Power correlation values vs. input power

### C. Measurement results

The test environment is like this: Input *I*, *Q* data was downloaded from Matlab to Agilent ESG to provide complex CDMA signals. The same Intersil's PA was used for testing. The output signal was downconverted to baseband and collected by Agilent PSA and VSA, which have large enough bandwidth and dynamic range to ignore their nonlinearity contributions.

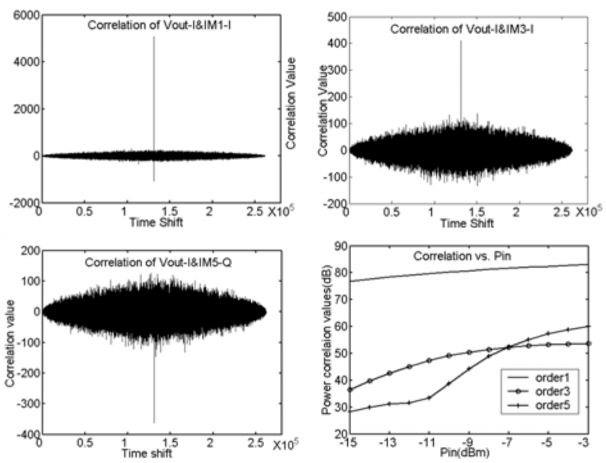


Fig.9. Correlation of different orders (a-c) @ $P_{in}$=-6dBm and power correlation values vs. $P_{in}$ (d)

In Fig.9 (a-c), the first, third and fifth order correlation values at different time shift positions are given. The power correlation values vs. swept input power are shown in Fig.9 (d). These curves have a similar trend as fundamental, IM3 and IM5 changes vs. input power in the two-tone measurement.

## III. Summary and Outlook

Simple correlation techniques have been used to estimate nonlinearity. Simulation results and comparison with single and two-tone test show promise of this method. The technique is potentially applicable to the measurement of amplifier nonlinearity in real time during amplifier operation. It is potentially carried out with simple circuitry. Extensions of the approach to include assessment of amplifier memory effects should also be possible.


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TABLE I Simulation Results

| Polynomial Coeff. | Without Filter | | | | | With Filter | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Equivalent Coeff. | $L=2^{20}$ | | $L=2^{15}$ | | Equivalent Coeff. | $L=2^{20}$ | | $L=2^{15}$ | |
| | | Mean | Std | Mean | Std | | Mean | Std | Mean | Std |
| $a_1$=0.812 | $\overline{a}_1$=1.5172 | 1.5165 | 2.5e-3 | 1.5152 | 2.1e-2 | $\overline{a}_1$=9.0081 | 9.0462 | 7.4e-2 | 9.0827 | 4.7e-1 |
| $a_3$=0.032 | $\overline{a}_3$=0.4080 | 0.4085 | 3.6e-3 | 0.4080 | 2.2e-2 | $\overline{a}_3$=1.4660 | 1.4985 | 4.8e-2 | 1.7024 | 2.9e-1 |
| $a_5$=0.0012 | $\overline{a}_5$=0.0012 | 0.0012 | 2.9e-5 | 0.0012 | 1.9e-4 | $\overline{a}_5$=0.0012 | 0.0025 | 1.9e-4 | 0.0125 | 1.1e-3 |